Mathematical League of University of Lodz

Series II 23/24

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address: piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 15.01.24.

Exercise 1. *A pawn starts at the point* $(0,0)$ *on the plane and makes a sequence of jumps. If the position of the pawn is* (x,y)*, where* $x,y \in \mathbb{Z}$*, then after a jump its position is of the form* $(x+n, y+k)$*, where* $n,k \in \mathbb{Z}$ *and* $|n|+|k|=5$*. What is the smallest number of jumps needed for the pawn to reach the point (2024, 2024)?*

Exercise 2. *Prove that for any prime number* p *there exist at most* 2 *natural numbers* n *for which* $p2^n+1$ *is a square of a natural number.*

Exercise 3. *A square matrix* A *is called magic if sums of elements in each row, in each column and in both diagonals are equal. Prove that*

$$A = \begin{bmatrix} s+x & s-x+y & s-y \\ s-x-y & s & s+x+y \\ s+y & s+x-y & s-x \end{bmatrix}$$

for some $s,x,y \in \mathbb{R}$*. Show that if* A *is an invertible magic matrix, then* A^{-1} *is also magic. Moreover, show that* A *is invertible if and only if* $x^2 \neq y^2$*.*