UNIC Mathematics Summer School, June 2026

Topic: Lie groups
Instructor: Prof. Ben McKay , University College Cork

1. **Course description**: We assume that the student has seen the definitions of manifolds, vector fields, and differential forms. We will focus on Lie groups as symmetry groups of differential equations.

2. **Course goals:**

   - To learn to apply Lie groups in working with differential equations.
   - To see how the structure of Lie groups relates to their application.

3. **Timetable**

Date & Time	Content
	Virtual phase
9(10).06.2026 (2 hrs) exact time to be announced later (TBA)	**Lecture 1: Review of Course Material** Presentation of the subject of the course and its connection with other fields of mathematics. Overview of the students' mathematical background necessary for the understanding of the lectured material.
16(17).06.2026 (2 hrs) exact time TBA	**Lecture 2: Introduction to the Topic** Defintions and examples of Lie groups and their actions arising in geometry and classical mechanics.
	Physical phase at Lodz University.
22-27.06.2026 (2 hrs) exact time TBA	**Lecture I:** Lie groups, left invariant vector fields, Lie brackets, exponentials, Lie algebras
22-27.06.2026 (2 hrs) exact time TBA	**Lecture II:** Actions, orbits, stabilizers, homogeneous spaces, quotients, integration by quadrature.
22-27.06.2024 (2 hrs) exact time TBA	**Lecture III:** Simplicity, roots, compact forms, complex forms, classification of semisimple Lie groups
22-27.06.2024 (2 hrs) exact time TBA	**Lecture IV:** Closed subgroups, topological groups, fundamental groups, integrating Lie algebra morphisms

Recommended reading/textbook list for the course:

- Robert Bryant, *An introduction to Lie groups and symplectic geometry*, https://sites.math.duke.edu/~bryant/ParkCityLectures.pdf
- John Stillwell, **Naive Lie Theory**

4. Mode of delivery:

- Students need to read the materials and complete the problem assignments.
- Virtual Seminars via Zoom or Microsoft Teams.
- Face-to-face classes at Lodz University.
- Virtual and face to face group meetings

5. Period: June 2026

Virtual phase: 9(10).06 and 16(17).06.2026
Physical phase: 21.06 - 28.06.2026

6. Students: Open to undergraduate students who have received at least 90 ECTS credits in mathematical courses or for 1st year graduate students.

7. Workload:

- Independent work (Reading materials, homework problems) 20 h
- Lectures 12 h (Virtual 4 h, Face to face 8 h)
- Total number of hours: 32 h

8. Assessment methods:

In order to pass this course, the following assessment criteria will be used:
- Compulsory participation in the virtual and physical lectures
- Compulsory reading and homework assignments

Grading: Pass/Fail or Satisfactory/Unsatisfactory

9. Teaching language: English

10. ECTS: 5 in total together with the other three topics:
- Lie groups
- Finite quantum groups
- Spectral graph theory
- C* algebras
- in the Summer School. Total workload of the four topics is 4x32=128 h.

Learning Outcomes

- learn to employ symmetry groups in physics and mathematics
- learn to integrate infinitesimal actions
- learn to use the classification of simple Lie groups to understand group structure

Topic: Finite quantum groups
Instructor: Pekka Salmi, University of Oulu

1. **Course description**:
Quantum groups generalise classical groups and, for example, allow the extension of Pontryagin duality to the non-commutative setting. To keep the prerequisites manageable, the course focuses on finite quantum groups. The definition of a finite quantum group is based on Hopf *-algebras. We will cover basic examples arising from groups, as well as genuinely quantum examples. The theoretical focus is on the duality of finite quantum groups. The duality is closely related to the Fourier transform and uses the Haar integral as a tool.

2. **Course goals:**
To provide an introduction to quantum groups and their duality in an accessible setting. After completing the course (and with additional prerequisites) students may proceed to the more general theories of compact or locally compact quantum groups.

3. **Timetable**

Date & Time	Content
Virtual phase	
9(10).06.2026 (2 hrs) exact time to be announced later (TBA)	**Lecture 1: Review of Course Material** Presentation of the subject of the course and its connection with other fields of mathematics. Overview of the students' mathematical background necessary for the understanding of the lectured material.
16(17).06.2026 (2 hrs) exact time TBA	**Lecture 2: Introduction to the Topic** Groups as algebras: the group algebra and the function algebra, definition of Hopf algebra, more examples
Physical phase at Lodz University.	
22-27.06.2026 (2 hrs) exact time TBA	**Lecture I:** Definition and examples of finite quantum groups, properties of the counit and the antipode
22-27.06.2026 (2 hrs) exact time TBA	**Lecture II:** The Haar integral of a finite quantum group
22-27.06.2024 (2 hrs) exact time TBA	**Lecture III:** Duality theory of finite quantum groups
22-27.06.2024 (2 hrs) exact time TBA	**Lecture IV:** Connection to the Fourier transform, moving beyond the finite case

Recommended reading/textbook list for the course:

J. Kustermans & L. Tuset: A survey of C*-algebraic quantum groups, part I, Irish Math. Soc. Bull. No. 43 (1999), 8-63.
T. Timmermann: An invitation to quantum groups and duality. From Hopf algebras to multiplicative unitaries and beyond. European Mathematical Society (EMS), Zürich, 2008.

4. **Mode of delivery:**

   - Students need to read the materials and complete the problem assignments.
   - Virtual Seminars via Zoom or Microsoft Teams.
   - Face-to-face classes at Lodz University.
   - Virtual and face to face group meetings

5. **Period**: June 2026

   Virtual phase: 9(10).06 and 16(17).06.2026
   Physical phase: 21.06 - 28.06.2026

6. **Students:** Open to undergraduate students who have received at least 90 ECTS credits in mathematical courses or for 1st year graduate students.

7. **Workload**:

   - Independent work (Reading materials, homework problems) 20 h
   - Lectures 12 h (Virtual 4 h, Face to face 8 h)
   - Total number of hours: 32 h

8. **Assessment methods:**

   In order to pass this course, the following assessment criteria will be used:
   - Compulsory participation in the virtual and physical lectures
   - Compulsory reading and homework assignments

   Grading: Pass/Fail or Satisfactory/Unsatisfactory

9. **Teaching language:** English

10. **ECTS:** 5 in total together with the other three topics:
   - Lie groups
   - Finite quantum groups
   - Spectral graph theory
   - C* algebras
   - in the Summer School. Total workload of the four topics is 4x32=128 h.

Learning Outcomes

After completing the course, the student:
- can explain the definition of a finite quantum group and describe how it generalises classical groups,
- is able to work with the Haar integral in the context of finite quantum groups,
- recognises the role of duality theory and its relation to the Fourier transform,
- can follow and reproduce algebraic arguments relevant to the theory at this level, using knowledge from linear algebra and basic algebra.

UNIC Mathematics Summer School, June 2026

Topic: Spectral Graph Theory
Instructor: Prof. Ivica Nakić, University of Zagreb

1. **Course description**: We will explore the fundamental connections between graphs and linear algebra via the spectral data of associated adjacency and Laplacian matrices. We will develop the tools needed to prove the celebrated Cheeger's inequality, which relates the second eigenvalue of the Laplacian to graph expansion. We will then apply these ideas to analyse random walks and their mixing times, and conclude with a brief outlook on further topics such as expander graphs and spectral sparsification.

2. **Course goals:**
   - To introduce students to the interplay between linear algebra and graph theory, demonstrating how algebraic methods reveal hidden structures in combinatorial objects.
   - To develop an understanding of the Cheeger's inequality as a paradigmatic example of this interplay, showing how a spectral quantity (the second eigenvalue) governs a fundamental combinatorial property (graph expansion).

3. **Timetable**

Date & Time	Content
Virtual phase	
9(10).06.2026 (2 hrs) exact time to be announced later (TBA)	**Lecture 1: Linear Algebra** Spectral theorem for symmetric matrices, Rayleigh quotient, Perron-Frobenius theorem
16(17).06.2026 (2 hrs) exact time TBA	**Lecture 2: Graphs, Matrices, Probability** Graph definitions, adjacency and Laplacian matrices, quadratic form, expectation, Markov's inequality
Physical phase at Lodz University.	
22-27.06.2026 (2 hrs) exact time TBA	**Lecture I: Laplacian Spectrum, Bounds** The second eigenvalue and connectivity, Fiedler vector, adjacency eigenvalue bounds, a bound on the chromatic number, spanning trees
22-27.06.2026 (2 hrs) exact time TBA	**Lecture II: Cheeger's inequality – Part 1** Graph partitioning, isoperimetry, conductance, statement of Cheeger's inequality
22-27.06.2024 (2 hrs) exact time TBA	**Lecture III: Cheeger's inequality – Part 2** Proof of the easy direction, probabilistic method, proof of the hard direction
22-27.06.2024 (2 hrs) exact time TBA	**Lecture IV: Random Walks & Outlook** Mixing times, spectral gap, expanders, sparsification

Recommended reading/textbook list for the course:

- Daniel A. Spielman: *Spectral and Algebraic Graph Theory* (draft), available at http://cs-www.cs.yale.edu/homes/spielman/sagt
- Bogdan Nica: *A brief introduction to Spectral Graph Theory*, European Mathematical Society (2018), preliminary version available at https://arxiv.org/abs/1609.08072v1

4. **Mode of delivery:**

- Students need to read the materials and complete the problem assignments.
- Virtual Seminars via Zoom or Microsoft Teams.
- Face-to-face classes at Lodz University.
- Virtual and face to face group meetings

5. **Period**: June 2026

Virtual phase: 9(10).06 and 16(17).06.2026
Physical phase: 21.06 - 28.06.2026

6. **Students:** Open to undergraduate students who have received at least 60 ECTS credits in mathematical courses or for 1st year graduate students.

7. **Workload**:

- Independent work (Reading materials, homework problems) 20 h
- Lectures 12 h (Virtual 4 h, Face to face 8 h)
- Total number of hours: 32 h

8. **Assessment methods:**

In order to pass this course, the following assessment criteria will be used:
- Compulsory participation in the virtual and physical lectures
- Compulsory reading and homework assignments

Grading: Pass/Fail or Satisfactory/Unsatisfactory

9. **Teaching language:** English

10. **ECTS:** 5 in total together with the other three topics:
- Lie groups
- Finite quantum groups
- C* algebras
  in the Summer School. Total workload of the four topics is 4x32=128 h.

Learning Outcomes

Students will be able to do upon completion:

- Explain the information revealed by the eigenvalues and eigenvectors of the adjacency matrix and Laplacian matrix.
- Compute the spectra of simple families of graphs and interpret the results in terms of graph structure.
- Prove the Cheeger's inequality and demonstrate how the Fiedler vector can be used to construct a cut with provably low conductance.
- Analyze the mixing time of a random walk on a graph using the relationship between the second eigenvalue of the Laplacian and the spectral gap of the transition matrix.

UNIC Mathematics Summer School, June 2026

- **Topic:** C* algebras

Instructor: Prof. Hanna Podsędkowska , Lodz University

1. **Course description**: The aim of the course is to present some elements of the theory of C* algebras such as basics of Banach algebras, the spectrum and the spectral radius, positive elements of C*-agebras, ideals in C*-algebras, *-homomorphisms, positive linear functionals, the Gelfand-Naimark Representation
2. **Course goals:** To provide the students with basic knowledge of the theory C*- algebras allowing them to study more advanced topics such as e.g. theory of operator algebras

3. **Timetable**

Date & Time	Content
Virtual phase	
9(10).06.2026 (2 hrs) exact time to be announced later (TBA)	**Lecture 1: Review of Course Material** Presentation of the subject of the course and its connection with other fields of mathematics. Overview of the students' mathematical background necessary for the understanding of the lectured material.
16(17).06.2026 (2 hrs) exact time TBA	**Lecture 2: Introduction to the Topic** Basics of Banach algebras: ideals, homomorphisms, spectrum, Gelfand-Mazur theorem, spectral mapping theorem
Physical phase at Lodz University.	
22-27.06.2026 (2 hrs) exact time TBA	**Lecture I:** C* algebra, involution, self adjoint element, normal and unitary element, projection,*homomorphism - definitions and spectral properties.
22-27.06.2026 (2 hrs) exact time TBA	**Lecture II:** Functional calculus on C*algebra, * homomorphism between C*algebras and related properties of spectum
22-27.06.2024 (2 hrs) exact time TBA	**Lecture III:** Order structure on C* algebra Functionals on C*algebra
22-27.06.2024 (2 hrs) exact time TBA	**Lecture IV:** GNS representation

4. **Recommended reading/textbook list for the course:**

Kadison, R. V. & Ringrose, J. R. (1983). ***Fundamentals of the Theory of Operator Algebras. Volume I: Elementary Theory.***

Murphy, G. J. (1990). *C-Algebras and Operator Theory**. New York, NY: Academic Press.

5. **Mode of delivery:**

    - Students need to read the materials and complete the problem assignments.
    - Virtual Seminars via Zoom or Microsoft Teams.
    - Face-to-face classes at Lodz University.
    - Virtual and face to face group meetings

6. **Period**: June 2026

    Virtual phase: 9(10).06 and 16(17).06.2026
    Physical phase: 21.06 - 28.06.2026

7. **Students:** Open to undergraduate students who have received at least 90 ECTS credits in mathematical courses or for 1st year graduate students.

8. **Workload**:

    - Independent work (Reading materials, homework problems) 20 h
    - Lectures 12 h (Virtual 4 h, Face to face 8 h)
    - Total number of hours: 32 h

9. **Assessment methods:**

    In order to pass this course, the following assessment criteria will be used:
    - Compulsory participation in the virtual and physical lectures
    - Compulsory reading and homework assignments

Grading: Pass/Fail or Satisfactory/Unsatisfactory

10. **Teaching language:** English

11. **ECTS:** 5 in total together with the other three topics:
- Lie groups
- Finite quantum groups
- Spectral graph theory
- C* algebras
- in the Summer School. Total workload of the four topics is 4x32=128 h.

Learning Outcomes

1. Demonstrate an understanding of the structure and spectral theory of C*-algebras, including involution, self-adjoint, normal and unitary elements, projections, *-homomorphisms, functional calculus, and the behavior of the spectrum under morphisms.

2. Explain and apply the theory of positive elements and functionals on C*-algebras

3. Construct and interpret the GNS representation associated with a given functional, and understand how it provides a canonical passage from an abstract C*-algebra to a concrete operator representation on a Hilbert space, enabling the use of operator-theoretic and Hilbert space methods in the analysis of C*-algebras.